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ESTIMATION OF DEFAULT RISK BY STRUCTURAL MODELS: THEORY AND LITERATURE

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Abstract

Default risk or Credit risk, is the likelihood of a firm losing money if a business partner defaults. If the liabilities are not met under the terms of the contract, the firm may default, resulting in the loss of the company. The most common way to estimate the default risk is by using default models. Default models employ market data to model the occurrence of a default event. These models have evolved into two distinct types of models: structural and reduced form models. Structural models measure the likelihood of a company defaulting based on its assets and liabilities. If the market worth of a company's assets is less than the debt it owes, it will default. Reduced form models often assume an external cause of default, such as a Poisson jump process, which is driven by a stochastic process. They model default as a random event with no regard for the balance sheet of the company. In this paper we use structural models to estimate the default risk by comparing the Distance to Default (DD) and the probability of default (PD) generated by two structural models: Merton and Moody's Kealhofer, McQuown, and Vasicek (MKMV) models of credit risk based on default probabilities generated from information in the equity market. Results show that the MKMV model compares well with the Merton model and performs a little bit better in some circumstances.

Keywords: *Default Risk, Structural Models, Reduced form Models, Merton Model, MKMV Model.*

1. INTRODUCTION

The uncertainty about a company's ability to service its debts and commitments is known as credit risk or default risk. It is the risk of a loss occurring as a result of a borrower's failure to repay a loan or meet contractual obligations. It refers to the likelihood of a corporation losing money if a business partner defaults. If the liabilities are not met under the terms of the contract, the firm may default, resulting in the loss of the company. Credit, commerce, and investment operations, as well as the payment system and trade settlement, all result in liabilities. Credit risk modeling is difficult due to the fact that company default is not a common occurrence and usually comes unexpectedly. However, when a creditor defaults, it frequently results in significant losses that cannot be predicted in advance, therefore effectively measuring and managing credit risk can reduce the severity of a loss (Mišanková et al. 2014). Credit risk default models are divided into two categories: structural and reduced form models. Structural models are used to measure the likelihood of a company defaulting based on its assets and liabilities. If the market worth of a company's assets is less than the debt it owes, it will default. Reduced form models typically assume an exogenous cause of default, such as a Poisson jump process, which is driven by a stochastic process. They model default as a random event with no regard for the balance sheet of the company. This paper examines the use of structural models to assess default risk, focusing on the work of Merton (1974) and Crosbie and Bohn's KMV model (2003).

2. STRUCTURAL MODELS

Structural models assess the structure of the company's capital and are based on the company's value. Merton (1974) pioneered structural models, which use the Black-Scholes option pricing framework to characterize default behavior. They are used to figure just how likely a company is to default based on the value of its assets and liabilities. They make the assumption that they have complete knowledge of a company's assets and liabilities, leading in a predicted default time. These models suggest that default risks arise when the value of a company's assets falls below its outstanding debt at the maturity date (Saunders and Allen 2002). These models are designed to show a direct link between default risk and capital structure. The fundamental disadvantage of this strategy is that it ignores the market value of a company's assets and treats debt as an option on those assets, and the default event is predictable (Chatterjee, 2015).

2.1 The Merton model

Robert Merton devised a model in 1974 for analyzing a company's structural credit risk by modeling its equity as a call option on its assets. The Merton model, which employs the Black-Scholes-Merton option pricing methodologies, is structural in that it establishes a link between default risk and the firm's asset (capital) structure. The book value of a company's equity E, total assets A, and total debts D of face amount K (strike price) maturing at time T, are all recorded on the balance sheet. The link between these values is established by the equation (Wang 2009):

$$A = E + D \quad (1)$$

A debt maturity T is chosen such that all debts are mapped into a zero-coupon bond. When $A_T > K$, the company's debt holders will be paid the full amount K, and shareholders' equity still has value $A_T - K$. On the other hand, the company defaults on its debt at T if $A_T < K$. In this case, debt holders will have first claim to the residual asset, A_T and shareholders will be left with nothing. The equity value at time T can be written as:

$$E_T = \max\left(A_T - K, 0\right)$$

(2)

This is the payoff of a European call option written on underlying asset A with a strike price of K and a maturity of T. Provided appropriate modeling assumptions are specified, the well-known Black-Scholes option pricing formulas can be used. Assume that the asset value is determined by a geometric Brownian motion (GBM) process with risk-neutral dynamics as defined by the stochastic differential equation:

$$dA = rAdt + \sigma_A AdW \quad (3)$$

where W is a standard Brownian motion under risk-neutral measure, *r* is the continuously compounded risk-free interest rate, and σ_A is the asset's return volatility. Under the risk-neutral measure, A rises at the risk-free rate and has drift *r* in (3), implying that corporate assets are continuously tradable. When the Black-Scholes formula is applied to European call options, we obtain the following equation:

$$E = AN\left(d_1\right) - Ke^{-rT} N\left(d_2\right)$$

(4)

where $N\left(.\right)$ is the $N\left(0,1\right)$ cumulative distribution probability function, with the quantities d_1 and d_2 given by:

$$d_1 = \frac{\ln\left(\frac{A}{K}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}},$$

(5)

$$d_2 = \frac{\ln\left(\frac{A}{K}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} = d_1 - \sigma_A\sqrt{T} \quad (6)$$

The debt's value is calculated by $A - E$. The risk-neutral likelihood of the corporation defaulting on its debt is $N\left(-d_2\right)$. Here, a credit default at time T is triggered by the occurrence where shareholders' call option matures out-of-money, with the following risk-neutral probability:

$$P\left(A_T < K\right) = N\left(-d_2\right), \quad (7)$$

By extracting the underlying market price of risk, this can occasionally be translated into a real-world probability. When asset level and return volatility (A and σ_A) are available for provided T, K and r, this allows us to solve for credit spread. One typical method of obtaining A and σ_A is to use another geometric Brownian motion model for equities prices E and use Ito's Lemma to demonstrate that instantaneous volatilities satisfy:

$$A\sigma_A \frac{\partial E}{\partial A} = E\sigma_E \quad (8)$$

using Black-Scholes equation, it can be shown that $\frac{\partial E}{\partial A} = N(d_1)$, then (8) we becomes:

$$A\sigma_A N(d_1) = E\sigma_E \quad (9)$$

The price of an equity E and the volatility σ_E of its return are observed in the equity market. Finally, (4) and (9), can be solved simultaneously for A and σ_A.

2.1.1 Calculation of Distance to Default (DD) by Merton Model

After determining the values of A and σ_A, we must calculate the distance to default (DD), which is defined as the number of standard deviations between the expected asset value at maturity T and the debt threshold D:

$$DD = \frac{\log A + \left(\mu_A - \sigma_A^2/2\right)T - \log(D)}{\sigma_A\sqrt{T}}$$

(10)

The expected return on the assets, which can be equal to the risk-free interest rate or any other value based on expectations for that firm, is the drift parameter μ_A. *DD* is the basis of credit evaluation. It is a standard index reflecting the company's credit quality, which can be compared for different companies and for different periods of time. The greater the value of *DD*, the more likely the company is to repay debts in due time, as a consequence the defaults will be less and the credit will be better (Chen et al., 2010).

2.1.2 Calculation of Probability of Default (PD) by Merton Model

The likelihood of default (PD) is the chance that the asset value will fall below the debt threshold at the end of the time horizon T and is given by:

$$PD = 1 - N(DD) = N(-DD) \quad (11)$$

The DD scaled by asset volatility reflects how far a firm's asset value is from the value of obligations that would trigger a default.

2.1.3 Estimation of DD and PD from Federal Reserve Economic Data by Merton model

Table 1 Shows the distribution of short term liabilities (STL), long term liabilities (LTL), and total asset values recorded from Federal Reserve Economic Data. Time *(T)* is the time in years where these data were recorded. We have taken a period of ten years from 2011/10/01 to 2020/10/01.

Table 1. Short and long term liabilities, average debts and total asset values

Time (T)	2011/10/01	2012/10/01	2013/10/01	2014/10/01	2015/10/01	2016/10/01	2017/10/01	2018/10/01	2019/10/01	2020/10/01
STL	3810	3829	3813	4177	5900	4336	3705	3585	4775	6003
LTL	16487	16947	19431	22299	30692	32037	29130	29690	28792	29921
Asset(V)	173063	171211	191450	205093	203037	198507	201953	211339	228884	253764

Source (Federal Reserve Economic Data, https://fred.stlouisfed.org, https://fredhelp.stlouisfed.org)

Table 2 shows the distances to default *(DD)* and probabilities of default *(PD)* calculated from **Table 1**. Averege asset value *A* and debt (liabilities) are used to calculate the Distance to Default *(DD)* in equation (14). *DD* is used to calculate the probability of default *PD* given by equation (15). From the table**,** we see that, *DD* calculated from short term liabilities (DD_{STL}) give larger values of *DDs* comparing to *DDs* produced by the long term liabilities (DD_{LTL}) indicating much stability to the company using short term liabilities. Also probalities of default generated by short term liabilities (PD_{STL}) are smaller than the *PDs* generated by long term liabilities (PD_{LTL}), indicating that short term liabilities give stable environments to the firm to default compared to those generated by the long term liabilities. This shows that, with long term liabilities, the firm has more risk to default compared to the short term liabilities.

Table 2. DD and PD from Table 1 by Merton Model

Time (T)	1	2	3	4	5	6	7	8	9	10
DD_{STL}	19.1860	13.5666	11.0771	9.5930	8.5803	7.8327	7.2516	6.7833	6.3953	6.0672
PD_{STL}	0.0	0.0	0.0	0.0	0.0	2.4e-15	2.0e-13	5.9e-12	8.0e-11	6.5e-10
DD_{LTL}	10.3847	7.3431	5.9956	5.1923	4.6442	4.2395	3.9250	3.6715	3.4616	3.2839
PD_{LTL}	0.0	1.0e-13	1.0e-09	1.0e-07	1.7e-06	1.1e-05	4.3e-05	0.0001	0.0003	0.0005

Figure 1 shows the distances to default (*DD)* from the debt maturity time (*T*) in years. The plot shows that the *DDs* and time (*T*) are inversely proportionally related. The *DD* decreases with increase in time to maturity (*T*). This means that, as time to maturity increases in years, the firm's stability to default decreases, hence the firm is subject to default as time to maturity increases.

Figure 1. Distances to Default from the debt maturity time in years

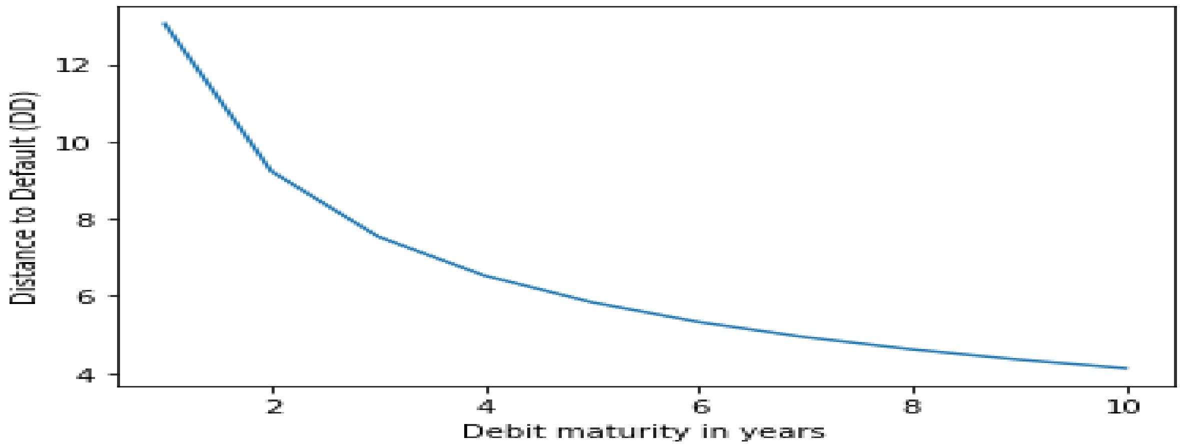
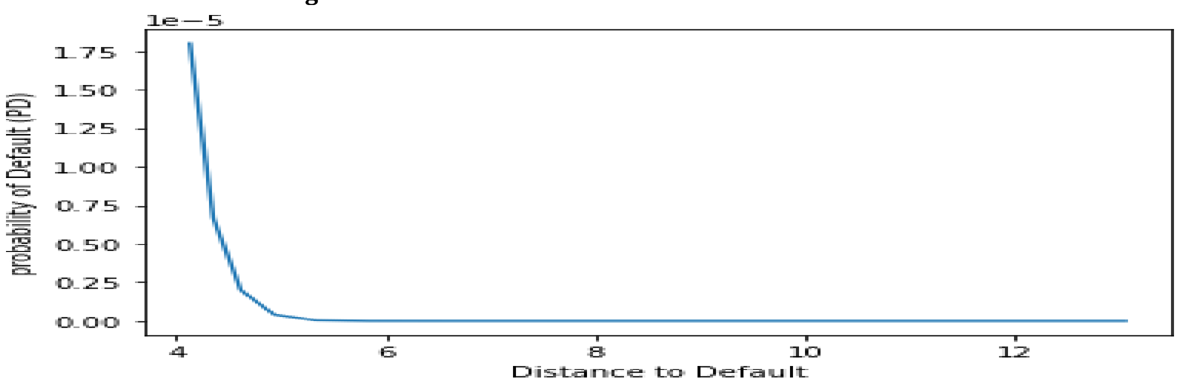


Figure 2 shows the Probabilities of Default (*PD*) from distances to default (*DD*). The plot shows that the *PDs* and *DD* are inversely proportionally related. The *PDs* decreases with increase in *DDs*. This means that, the lower the *PDs*, the higher the *DDs* and hence more stable is the the firm.

Figure 2. Probabilities of Default the Distance to Default



2.2 Moody's KMV (MKMV) Model

Kealhofer, McQuown and Vasicek (*KMV*) model developed by the *KMV* Company is based on the Merton Option Pricing Theory (Merton 1974). It is a set of conceptual frameworks to estimate the default probability of a company. The *KMV* model assumes that the company will default when the company's asset value is less than liabilities. According to the basic idea of Merton model, the *KMV* model regards the company's equity value as the call option, which considers asset value as the underlying asset and the debt as the exercise price. *KMV* model is based on the structural approach to calculate Expected default frequency (*EDF*) as a forward-looking measure of actual probability of default than the normal Probability of Default (*PD*) measured by the Merton approach. *EDF* is firm specific. The approach is best when applied to publicly traded companies, where the value of equity is determined by the stock market. The market information contained in the firm's stock price and balance sheet are translated into an implied risk of default. According to *KMV*'s empirical studies, log-asset returns confirm quite well to a normal distribution, and σ_A stays relatively constant. *KMV* approach outlines three steps to derive the actual probabilities of default:

i. Estimation of the market value and volatility of the firm's asset
ii. Calculation of the distance to default, an index measure of default risk
iii. Scaling of the distance to default to actual probabilities of default using a default database.

In 2002, Moody's Corporation acquired *KMV*, a leading provider of quantitative credit analysis tools to lenders, investors, and corporations, and hence the name *MKMV* (Voloshyn 2015). The relationship between equity value and asset value is described by the Black-Scholes option pricing formula:

$$E = AN(d_1) - De^{-r\tau}N(d_2) = f(A, \sigma_A, r, D, \tau) \quad (12)$$

Where E denotes the equity value, A denotes the asset value, D denotes the default point, σ_A denotes the asset value volatility, r denotes the risk free rate, τ denotes the debt maturity, $N(\cdot)$ denotes the standard normal cumulative distribution function. In Equation. (17), d_1 and d_2 can be obtained as follows:

$$d_1 = \frac{\ln(A/D) + (r + \sigma_A^2/2)\tau}{\sigma_A\sqrt{\tau}} \quad (13)$$

$$d_2 = \frac{\ln(A/D) + (r - \sigma_A^2/2)\tau}{\sigma_A\sqrt{\tau}} = d_1 - \sigma_A\sqrt{\tau} \quad (14)$$

There are two unknown parameters A and σ_A in Equation. (12) that need to be solved. This can be achieved by introducing the relationship between the equity value volatility (σ_E) and the asset value volatility (σ_A) by (Nazeran and Dwyer 2015):

$$\sigma_E = \frac{AN(d_1)}{E}\sigma_A = g(A, \sigma_A, r, D, \tau) \quad (15)$$

After substituting Equation (12) and (13) into Equation (15), we can derive that σ_E is the function of A, σ_A, r, D and τ. We set a system of equations from Equations (12) and (15) where A and σ_A are unknown parameters by:

$$\begin{cases} f(A, \sigma_A) - E = 0 \\ g(A, \sigma_A) - \sigma_E = 0 \end{cases} \quad (16)$$

Then we calculate the Jacobian Matrix of the functions. According to the Newton-iterative method, which builds iteration through Taylor expansion, we get Equation (17) as:

$$\begin{pmatrix} A^{(k+1)} \\ \sigma_A^{(k+1)} \end{pmatrix} = \begin{pmatrix} A^{(k)} \\ \sigma_A^{(k)} \end{pmatrix} - \begin{pmatrix} \frac{\partial f}{\partial A} & \frac{\partial f}{\partial \sigma_A} \\ \frac{\partial g}{\partial A} & \frac{\partial g}{\partial \sigma_A} \end{pmatrix}^{-1} \begin{pmatrix} f\left(A^{(k)}, \sigma_A^{(k)}\right) - E \\ g\left(A^{(k)}, \sigma_A^{(k)}\right) - \sigma_E \end{pmatrix},$$

(17)

where the appropriate initial values (A_0, σ_A^0) in Newton iteration are set by use of the trial and error method.

2.2.1 Calculation of Default Point (DPT) (Liability of the firm)

MKMV Company found that the companies generally do not default when their assets value is up to the book value of total liabilities. When the company defaults, the asset value is generally between the short term liabilities and the book value of total liabilities (Crosbie and Bohn 2003):

$$DPT = STL + k \times LTL, \quad 0 \le k \le 1,$$

(18)

where DPT denotes default point, STL denotes current liabilities, and LTL denotes long-term liabilities. After a great deal of observations to the default companies, *MKMV* Company found that the most frequent default point is at $k = 0.5$, and the predictive accuracy of model is sensitive to the changes of default point (Crosbie and Born 2003, Kealhofer and Bohn, 2001).

2.2.2 Calculation of Liability maturity (τ)(Time)

Because of the limited availability of data and information, the calculation time is set for one year to predict the credit risk in next year. The assumption here is that, the firm's liabilities will be matured in the time of one year. That is, the time $\tau = T - t = 1$.

2.2.3 Calculation of risk free rate (r)

For the risk free rate, we adopt one-year time deposit rate. But since this rate is fluctuating from month to month in last few years, we take the average of the 12 month's interest rates in the forecasting year in order to produce more accurate results.

2.2.4 Calculation of Distance to Default (DD) by MKMV Model

When A and σ_A are given, the distance to default (DD) in indebted companies can be calculated (Crosbie and Born 2003). The default point term-structure (DPT) (the default barrier at different points in time in the future) is determined empirically. $MKMV$ combines market asset value, asset volatility, and the default point term-structure to calculate a Distance-to-default (DD) term-structure. This term- structure is translated to a physical default probability using an empirical mapping between DD and historical default data (Crosbie and Bohn 2003, Kealhofer 2003, and Vasicek 1984). The equation for *DD* is given by:

$$DD = \frac{E(A_T) - DPT}{\sigma_A}$$

$$= \frac{\ln\left(\frac{A}{DPT}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}}$$

(19)

where DPT is the default point, A is the current market value of the firm, μ_A is the expected net return on firm value and σ_A is the annualized firm value volatility. It is assumed that the asset value of the company follows the normal distribution, thus the default distance (DD) reflects the standard deviation from company's default. If asset value A falls below DPT at any point in time, then the firm is considered to be in

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default. The default probability generated by the *MKMV* implementation is called an Expected Default Frequency or *EDF* credit measure as described by Huang (2003). In the DD-to-EDF empirical mapping step, *MKMV* model estimates a term-structure of this default barrier to come up with a *DD* term structure that could be mapped to a default-probability term-structure. Then the company's expected default frequency (*EDF*) is given by:

$$EDF = 1 - N(DD) = N(-DD) \quad (20)$$

However, the assumption that the asset value is subject to normal distribution is questionable. The *MKMV* Company tries to obtain the empirical value of *EDF* rather than the theoretical value from models. They count the number of default companies with same *DD* in a year and evaluate, the empirical value of *EDF* as the ratio of the above counts to the total number of companies with the same *DD*. *DD* is normally taken as the basis of credit evaluation because it is a standard index reflecting the company's credit quality, which can be compared for different companies and for different periods of time. The greater the value of *DD*, the more likely the company will be able to repay debts in due time, as a consequence the defaults will be less and the credit will be better (Chen et al., 2010).

2.2.6 Estimation of DD and EDF from Federal Reserve Economic Data by MKMV model

Table 3 shows the short term liabilities (*STL*), long term liabilities (*LTL*) and total asset value (*A*) recorded from Federal Reserve Economic Data. The table also shows the default point (*DPT*) calculated from *STL* and *LTL* shown using different values of *k* (0, 0.3, 0.5 and 1) as shown in equation (18). The *DPT* calculated helps in the determination of Distance to Default (*DD*) and Expected Default Frequency (*EDF*) by *MKMV* approach.

Table 3. STL, LTL, DPT and Assets Data by MKMV

Time (T)	2011/10/01	2012/10/01	2013/10/01	2014/10/01	2015/10/01	2016/10/01	2017/10/01	2018/10/01	2019/10/01	2020/10/01
STL	3810	3829	3813	4177	5900	4336	3705	3585	4775	6003
LTL	16487	16947	19431	22299	30692	32037	29130	29690	28792	29921
$DPT_{k=0}$	3810	3829	3813	4177	5900	4336	3705	3585	4775	6003
$DPT_{k=0.3}$	8756.1	8913.1	9642.3	10866.7	15107.6	13947.1	12444.0	12492.0	13412.6	14979.3
$DPT_{k=0.5}$	12053.5	12302.5	13528.5	15326.5	21246	20354.5	18270	18430	19171	20963.5
$DPT_{k=1}$	20297	20776	23244	26476	36592	36373	32835	33275	33567	35924
Asset(V)	173063	171211	191450	205093	203037	198507	201953	211339	228884	253764

Source (Federal Reserve Economic Data, https://fred.stlouisfed.org, https://fredhelp.stlouisfed.org)

Table 4 shows the Distances to Default (*DDs*) and *EDFs* calculated from **Table 3** using default points (*DPTs*) taken at different values of k, i.e., $k = 0, 0.3, 0.5$ and 1. From **Table 4**, we see that the *DDs* and *EDFs* generated at *k* = 0 (ie., $DD_{k=0}$ and $EDF_{k=0}$), are the same as the *DDs* and *PDs* generated using short term liabilities (*STL)* by Merton model. The $DD_{k=0}$ give greater Distance to default values and lesser *EDFs,* indicating that, firms at $k = 0$, will have stronger stability with less risk to default. At $k = 1$, we have lesser values of *DDs* and greater values of *EDFs*, indicating that, firms at $k = 1$, will be much risky to default. The table also shows that, for smaller values of k (i.e., $0 \le k \le 0.5$), the firm generates greater *DD* values with lesser *EDFs*, ensuring the stability of the firm to default.

Table 4. DD and EDF from Table 3 by MKMV Model

Time (T)	1	2	3	4	5	6	7	8	9	10
$DD_{k=0}$	19.1860	13.5666	11.0771	9.5930	8.5803	7.8327	7.2516	6.7833	6.3953	6.0672
$DD_{k=0.3}$	14.1386	9.9975	8.1629	7.0693	6.3230	5.7720	5.3439	4.9987	4.7129	4.4710
$DD_{k=0.5}$	12.3722	8.7485	7.1431	6.1861	5.5330	5.0509	4.6762	4.3742	4.1241	3.9124
$DD_{k=1}$	9.5911	6.7820	5.5374	4.7956	4.2893	3.9156	3.6251	3.3910	3.1970	3.0330

$EDF_{k=0}$	0.0	0.0	0.0	0.0	0.0	2.4e-15	2.0e-13	5.9e-12	8.0e-11	6.5e-10
$EDF_{k=0.3}$	0.0	0.0	1.1e-16	7.8e-13	1.3e-10	3.9e-09	4.5e-08	2.9e-07	1.2e-06	3.9e-06
$EDF_{k=0.5}$	0.0	0.0	4.6e-13	3.1e-10	1.6e-08	2.2e-07	1.5e-06	6.1e-06	1.9e-05	4.6e-05
$EDF_{k=1}$	0.0	5.9e-12	1.5e-08	8.1e-07	9.0e-06	4.5e-05	0.0001	0.0003	0.0007	0.0012

Figure 4 shows the *DDs* taken at $k = 0.5$, from debt maturity time in years. We can see from the figure that, *DDs* are inversely proportional to the maturity time. The *DDs* decrease with longer debt maturity time as seen in Merton model. This observation indicates that, the firm is more risky to default with longer debt maturity.

Figure 4. Distances to Default from debt maturity in years by *MKMV*

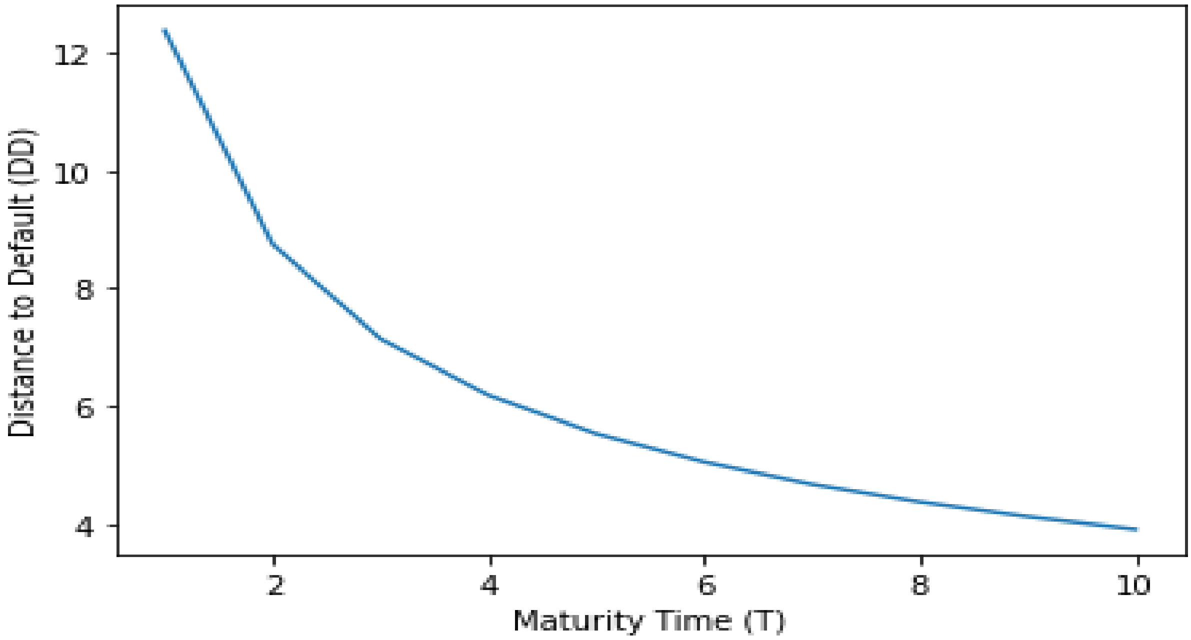

Figure 4 shows the *EDF* values from *DD*s. The plot shows that the *EDF* values decrease with increase in *DD* values. This indicates that, the firm becomes more stable as *DD* values increases. The larger the *DDs*, the stable is the firm to default.

Figure 5. Expected Default Frequences from Distances to Default by *MKMV*

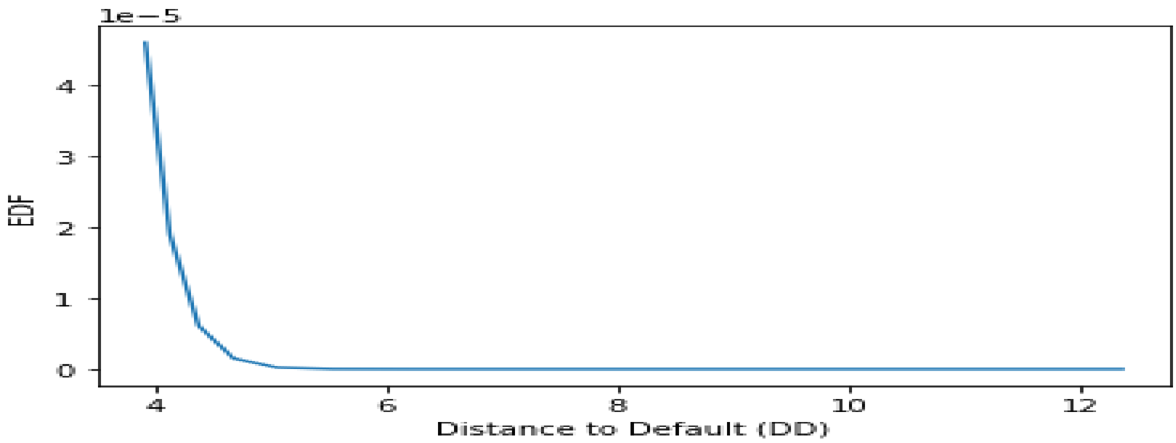

Table 5 shows the comparison between distances to default generated by Merton and *MKMV* models. The distances are calculated from short term liabilities (DD_{STL}) and long term liabilities (DD_{LTL}) using Merton model and using default points calculated at different values of k (i.e., $k = 0, 0.3, 0.5, 1$) by *MKMV* model. The

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table shows that, the *DDs* calculated using *STL* by Merton approach are equal to the *DDs* calculated by *MKMV* when $k=0$ at the default point. By *MKMV* approach, the table shows that, the *DDs* calculated using defaul points with k values less than or equal to 0.5 are greater compared to *DD* values generated by $k>0.5$. The best values are seen at $DD_{k=0.3}$, though the literature suggest $k=0.5$ to be considered on default points for *MKMV* approach. The *DD* values generated by *MKMV* at $k=0.5$ are on average greater than the *DD* values generated by using long term liabilities by Merton model.

Table 5. Comparison between *DDs* generated by Merton and *MKMV* models

Time (T)	1	2	3	4	5	6	7	8	9	10
DD_{STL}	19.1860	13.5666	11.0771	9.5930	8.5803	7.8327	7.2516	6.7833	6.3953	6.0672
DD_{LTL}	10.3847	7.3431	5.9956	5.1923	4.6442	4.2395	3.9250	3.6715	3.4616	3.2839
$DD_{k=0}$	19.1860	13.5666	11.0771	9.5930	8.5803	7.8327	7.2516	6.7833	6.3953	6.0672
$DD_{k=0.3}$	14.1386	9.9975	8.1629	7.0693	6.3230	5.7720	5.3439	4.9987	4.7128	4.4710
$DD_{k=0.5}$	12.3722	8.7485	7.1431	6.1861	5.5330	5.0509	4.6762	4.3742	4.1241	3.9124
$DD_{k=1}$	9.5911	6.7820	5.5374	4.7956	4.2893	3.9156	3.6251	3.3910	3.1970	3.0330

Table 6 shows the comparison between probabilities of default (*PD*) generated by Merton model and Expected default frequences (*EDF*) generated by *MKMV* model. The *PDs* are calculated from the short term liabilities(PD_{STL}) and long term liabilities (PD_{LTL}) using Merton model. The *EDFs* are calculated using *DDs* generated from default points taken at different k values, $EDF_{k=0}$, $EDF_{k=0.3}$, $EDF_{k=0.5}$ and $EDF_{k=1}$ by *MKMV* model. The table shows that, the *PDs* calculated using *STL* by Merton approach are equal to the *EDFs* calculated by *MKMV* when $k=0$ at the default point. On average, the table shows that, the *EDFs* generated by *MKMV* approach, give smaller values of probabilty to default compared to the *PDs* generated by Merton approch except at *EDFs* generated $k=1$. At $k \leq 0.5$, the *MKMV* approach give smaller values of probability to default compared to the Merton approch.

Table 6. Comparison between *PD* and *EDF* generated by Merton and *MKMV* respectively

Time (T)	1	2	3	4	5	6	7	8	9	10
PD_{STL}	0.0	0.0	0.0	0.0	0.0	2.4e-15	2.0e-13	5.9e-12	8.0e-11	6.5e-10
PD_{LTL}	0.0	1.0e-13	1.0e-09	1.0e-07	1.7e-06	1.1e-05	4.3e-05	0.0001	0.0003	0.0005
$EDF_{k=0}$	0.0	0.0	0.0	0.0	0.0	2.4e-15	2.0e-13	5.9e-12	8.0e-11	6.5e-10
$EDF_{k=0.3}$	0.0	0.0	1.1e-16	7.8e-13	1.3e-10	3.9e-09	4.5e-08	2.9e-07	1.2e-06	3.9e-06
$EDF_{k=0.5}$	0.0	0.0	4.6e-13	3.1e-10	1.6e-08	2.2e-07	1.5e-06	6.1e-06	1.9e-05	4.6e-05
$EDF_{k=1}$	0.0	5.9e-12	1.5e-08	8.1e-07	9.0e-06	4.5e-05	0.0001	0.0003	0.0007	0.0012

3. CONCLUSION AND SUGGESTION FOR FUTURE RESEARCH

In this paper we have compared the distances to default (*DDs*) and probabilities of default generated by two structural models in estimating the firm's default risk. These models are Merton model and the *MKMV* model. We generated *DDs* and *PDs* from information in the equity market using Federal Reserve Economic Data for both methods and compared their values. Using the Merton model, the *DDs* and *PDs* were calculated from short term liabilities (*STL*) and long term liabilities (*LTL*). Using *MKMV*, the *DDs* and *EDFs* were calculated using default points at different values of k (i.e., $k=0, 0.3, 0.5, 1$). Results show that, the *DDs* calculated using *STL* by Merton approach are equal to the *DDs* calculated by *MKMV* at $k=0$. For $0<k\leq 0.5$, the *DDs* generated by *MKMV* approach are larger than those generated by the Merton model using the long term liabilities. For $k>0.5$ the *DDs*, are smaller than than those generated by the Merton model. The best value for *MKMV* approach is seen at $k=0.3$, though the literature suggest $k=0.5$ to be considered on default points for *MKMV* approach. The *DD* values generated by *MKMV* at $k=0.5$ are on average greater than the *DD* values generated by using long term liabilities by Merton model.

The *PDs* are calculated from the short term liabilities(PD_{STL}) and long term liabilities (PD_{LTL}) using Merton model and the *EDFs* are calculated using *DDs* generated from default points taken at different k values, $EDF_{k=0}$, $EDF_{k=0.3}$, $EDF_{k=0.5}$ and $EDF_{k=1}$ by *MKMV* model. Results indicates that, the *PDs* calculated using *STL* by Merton approach are equal to the *EDFs* calculated by *MKMV* when $k=0$ at the default point. On average, results show that, the *EDFs* generated by *MKMV* approach, give smaller values of probabilty to default compared to the *PDs* generated by Merton approch except at *EDFs* generated at $k=1$. At $k \leq 0.5$, the *MKMV* approach give smaller values of probability to default compared to the Merton approch as the literature suggests. Generally, results show that the *MKMV* model compares well with the Merton model and performs a little bit better in some circumstances especially in probabilities of default.

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